

### Worked Solutions

#### Edexcel C3 Paper D

1. (a)  $\frac{(x+3)(x+4)}{x(x+3)} = \frac{x+4}{x}$  (3)

(b)  $\log_3 \left( \frac{x^2 + 7x + 12}{x^2 + 3x} \right) = 2$

$$\frac{x+4}{x} = 3^2$$

$$x+4 = 9x$$

$$x = \frac{1}{2} \quad (4)$$

2. (a)  $f(x) > 0$  (2)

(b)  $f\left(1\frac{1}{2}\right) = 2$

$$g(2) = 9 + k$$

$$9 + k = 14 \Rightarrow k = 5 \quad (4)$$

3. (a)  $\cos 4x = \cos 2(2x)$   
 $= 2 \cos^2 2x - 1$

$$= 2(2 \cos^2 x - 1)^2 - 1$$

$$= 8 \cos^4 x - 8 \cos^2 x + 1 \quad (4)$$

(b)  $8 \cos^2 x - 8 \cos^4 x - 1 = 0 \Rightarrow \cos 4x = 0$

$$4x = 90, 270, 450, 630$$

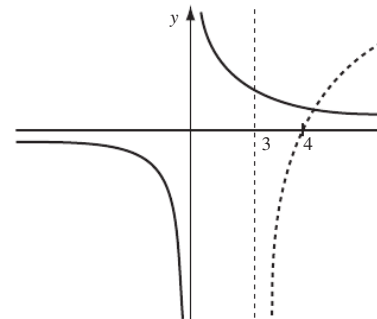
$$\therefore x = 22\frac{1}{2}, 67\frac{1}{2}, 112\frac{1}{2}, 157\frac{1}{2} \quad (3)$$

4. (a) (i)  $x^3 \cdot 3e^{3x} + 3x^2 \cdot e^{3x}$

(ii)  $\frac{\cos x \cdot 2 + 2x \cdot \sin x}{\cos^2 x}$

(b)  $\frac{dx}{dy} = -\sin y^2 \cdot 2y \quad \frac{dy}{dx} = -\frac{1}{2y \sin y^2}$

5. (a)



(b) curves cross once

(c)  $f(4) = \ln 1 - \frac{2}{4} = -\frac{1}{2}$

$f(5) = \ln 2 - \frac{2}{5} = 0.293$

} change in sign  $\Rightarrow$  root

(d)  $x_1 = 4.4918$

$x_2 = 4.5609$

$x_3 = 4.5504$

$x_4 = 4.5520$

$x = 4.55$  (2 d.p.)

6. (a)  $\cot x = \frac{4}{3}$ :

$$\operatorname{cosec}^2 x = 1 + \cot^2 x$$

$$= 1 + \frac{16}{9} = \frac{25}{9}$$

$$\operatorname{cosec} x = \pm \frac{5}{3}$$

$x$  is reflex,  $\therefore \operatorname{cosec} x = \frac{-5}{3}$

$$\left[ \begin{array}{l} \text{or } \tan x = \frac{3}{4} \\ \sin x = -\frac{3}{5}, \text{ } x \text{ is reflex} \end{array} \right]$$

(b)  $\sin x = \frac{-3}{5}$

so  $\cos 2x = 1 - 2\sin^2 x$

$$\Rightarrow \cos 2x = 1 - 2 \times \frac{9}{25}$$

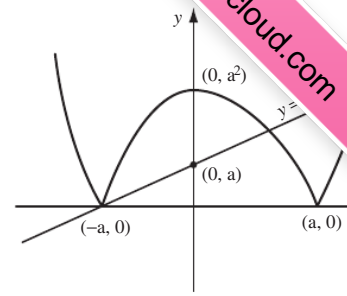
$$= \frac{7}{25}$$

(c)  $\tan x = \frac{3}{4}$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \left. \vphantom{\tan 2x} \right\} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{24}{7}$$

$$= \frac{2 \cdot \frac{3}{4}}{1 - \frac{9}{16}} \left. \vphantom{\tan 2x} \right\} = \frac{3}{2} = \frac{24}{7}$$

7. (a)



(b) see graph,  $y = 0$ ,  $x^2 = a^2 \quad \therefore x = \pm a$

(c)  $x^2 - a^2 = x + a$

$$x^2 - x - a^2 - a = 0$$

(d)  $-(x^2 - a^2) = x + a$

$$x^2 + x + a - a^2 = 0$$

(e)  $x^2 - x - 12 = 0 \quad (a = 3)$

$$(x - 4)(x + 3) = 0 \quad 2^{\text{nd}} \text{ pt. } (4, 7)$$

$$x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0 \quad 3^{\text{rd}} \text{ pt. } (2, 5)$$

$$8. \quad \frac{dx}{dt} = e^{-t}(-\sin t + \cos t) + (-e^{-t})(\cos t + \sin t) = -2e^{-t} \sin t$$

$$\frac{d^2x}{dt^2} = 2e^{-t} \sin t - 2e^{-t} \cos t$$

$$2e^{-t} \sin t - 2e^{-t} \cos t - 4e^{-t} \sin t + 2e^{-t}(\cos t + \sin t) = 0 \quad (6)$$

$$(a) \quad x = e^0(\cos 0 + \sin 0) = 1$$

$$\frac{dx}{dt} = -2e^{-0} \cdot \sin 0 = 0 \quad (2)$$

$$(b) \quad \sin t = 0 \Rightarrow t = 0, \pi, 2\pi \text{ etc.} \quad t = \pi \quad (2)$$

$$(c) \quad x = 1, x = e^{-\pi}(-1) = -e^{-\pi} \quad (2)$$

$$(d) \quad \frac{d^2x}{dt^2} = -2x - 2\frac{dx}{dt} = -2x$$

$$x = 1, \frac{d^2x}{dt^2} < 0 \quad \therefore \text{max.}$$

$$x = -e^{-\pi}, \frac{d^2x}{dt^2} > 0 \quad \therefore \text{min.} \quad (2)$$